GROUPS OF ORDER 72

SUMMARY

order	72
# abelian	6
# other decomposable	24
# other indecomposable*	20
TOTAL	50

72 = $\langle A^{72} \rangle$

72.02 = 36 × 2 = $\langle A^{36}, B^2 \rangle$

72.03 = 18 × 2^2 = $\langle A^{18}, B^2, C^2 \rangle$

72.04 = 24 × 3 = $\langle A^{24}, B^3 \rangle$

72.05 = 12 × 6 = $\langle A^{12}, B^6 \rangle$

72.06 = 6^2 × 2 = $\langle A^6, B^6, C^2 \rangle$

72.07 = D_8 × 3^3 = $\langle A^4, B^2, C^3, D^3, BA = A^{-1}B \rangle$

72.08 = Q_8 × 3^3 = $\langle A^4, B^2 = A^2, C^3, D^3, BA = A^{-1}B \rangle$

72.09 = D_8 × 9 = $\langle A^4, B^2, C^9, BA = A^{-1}B \rangle$

72.10 = Q_8 × 9 = $\langle A^4, B^2 = A^2, C^9, BA = A^{-1}B \rangle$

72.11 = D_6 × 6 × 2 = $\langle A^3, B^2, C^6, D^2, BA = A^{-1}B \rangle$

72.12 = 12.4 × 6

72.13 = D_6 × 12 = $\langle A^3, B^2, C^{12}, BA = A^{-1}B \rangle$

72.14 = 24.09 × 3

72.15 = $A_4 \times 6$

72.16 = 36.08 × 2

72.17 = 18.4×2^2

72.18 = 36.10 × 2

72.19 = 18.4 × 4

72.20 = $\langle A^3, B^3, C^8, CA = A^{-1}C, CB = B^{-1}C \rangle$
CLASS EQUATION: 72 = 1∗4 + 2∗16 + 9∗4 **#CC** = 24
ORDER EQUATION: 281∗2 = 2 + 3∗8 + 4∗2 + 6∗8 + 8∗36 + 12∗16
NORMAL SUBGROUPS:

H	**2**	**3**	**4**	**6**	3^2	**12**	**18×3**	**12 × 3**
G/H	**36.10**	**24.9**	**18.4**	**12.4**	**8**	D_6	**4**	**2**

× 4 **Z** = $\mathfrak{N}$ = **Φ** × 4 **G′** × 4

72.21 = $D_{18} \times 2^2$

72.22 = 36.12 × 2

72.23 = $D_{18} \times 4$

72.24 = $D_{9,8} = \langle A^9, B^8, BA = A^{-1}B \rangle$
CLASS EQUATION: 72 = 1∗4 + 2∗16 + 9∗4 **#CC** = 24
ORDER EQUATION: 479∗2 = 2 + 3∗2 + 4∗2 + 6∗2 + 8∗36 + 9∗6 + 12∗4 + 18∗6 + 36∗12
NORMAL SUBGROUPS:

H	**1**	**2**	**3**	**4**	**9**	**6**	**18**	**12**	**36**	**G**
G/H	**G**	**36.12**	**24.9**	**18.5**	**8**	**12.4**	**4**	D_6	**2**	**1**

Z = $\mathfrak{N}$ **G′** **Φ**

Sylow subgroups:
Maximal subgroups:

72.25 = 24.11 × 3

72.26 = 24.12 × 3

72.27 = 24.13 × 3

72.28 = 24.14 × 3

72.29 = ⟨A^4, B^2 = A^2, C^9, BA = A^−1B, CA = BC, CB = AB^−1C⟩

CLASS EQUATION: 72 = 1∗6 + 4∗12 + 6∗3 **#CC** = 21
ORDER EQUATION: 418∗2 = 2 + 3∗2 + 4∗6 + 6∗2 + 9∗24 + 12∗12 + 18∗24
NORMAL SUBGROUPS:

H	1	2	3	6	8.5	24.5	G
G/H	G	36.8	24.14	12.5	9	3	1

Z = ℜ = Φ G′

Sylow subgroups:
Maximal subgroups:

72.30 = 24.15 × 3

72.31 = D6^2 × 2 = ⟨A^3, B^2, C^3, D^2, E^2, BA = A^−1B, DC = C^−1D⟩

72.32 = 12.4 × D6

72.33 = ⟨A^2, B^3, C^4, D^2 = C^2, CA = A^−1C, DB = B^−1D⟩

CLASS EQUATION: 72 = 1∗2 + 2∗4 + 3∗4 + 9∗2 **#CC** = 12
ORDER EQUATION: ∗2 = 2∗19 + 3∗8 + 4∗12 + 6∗8 + 12∗24
NORMAL SUBGROUPS:

H	1	2	3	6	3^2	12.4	6 × 3	18.4	36.6	36.9	G
G/H	G	36.13	24.7	12.3	4 × 2	D6	2^2	4	2	2	1
	Z = ℜ = Φ	× 2	× 2	G′	× 2		× 2	× 2			

Sylow subgroups:
Maximal subgroups:

72.34 = 36.14 × 2

72.35 = ⟨A^3, B^3, C^8, CA = BC, CB = AC⟩

CLASS EQUATION: 72 = 1∗2 + 4∗4 + 9∗6 **#CC** = 12
ORDER EQUATION: ∗2 = 2 + 3∗8 + 4∗18 + 6∗8 + 8∗36
NORMAL SUBGROUPS:

H	1	2	3^2	6 × 3	36.10	G
G/H	G	36.14	8	4	2	1

Z = ℜ = Φ G′

Sylow subgroups:
Maximal subgroups:

72.36 = ⟨A^2, B^3, C^3, D^2, E^2, EB = $B^{-1}E$, EC = $C^{-1}E$, ED = ADE⟩

CLASS EQUATION: 72 = 1∗2 + 2∗17 + 18∗2 **#CC** = 21

ORDER EQUATION: ∗2 = 2∗21 + 3∗8 + 4∗18 + 6∗24

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	2^2	3^2	**6 × 2**	**6 × 3**	6^2	**36.9**	**G**
G/H	**G**	**36.9**	**24.11**	**18.4**	D_8	D_6	2^2	**2**	**2**	**1**

Z = ℜ = Φ × 4 **Z_2** × 4 **G′** × 2

Sylow subgroups:

Maximal subgroups:

72.37 = ⟨A^4, B^3, C^3, D^2, DA = $A^{-1}D$, DB = $B^{-1}D$, DC = $C^{-1}D$⟩

CLASS EQUATION: 72 = 1∗2 + 2∗17 + 18∗2 **#CC** = 21

ORDER EQUATION: ∗2 = 2∗37 + 3∗8 + 4∗2 + 6∗8 + 12∗16

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	**4**	**6**	3^2	**12**	**6 × 3**	**36.2**	**36.9**	**G**
G/H	**G**	**36.9**	**24.12**	**18.4**	**12.3**	D_8	D_6	2^2	**2**	**2**	**1**

Z = ℜ = Φ × 4 **Z_2** × 4 × 4 **G′** × 2

Sylow subgroups:

Maximal subgroups:

72.38 = ⟨A^3, B^3, C^4, $D^2 = C^2$, DA = $A^{-1}D$, DB = $B^{-1}D$, DC = $C^{-1}D$⟩

CLASS EQUATION: 72 = 1∗2 + 2∗17 + 18∗2 **#CC** = 21

ORDER EQUATION: 185∗2 = 2 + 3∗8 + 4∗38 + 12∗16

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	**4**	**6**	**6 × 2**	**6 × 3**	**12 × 3**	**36.10**	**G**
G/H	**G**	**16.09**	Q_{24}	**18.4**	$D_6 \times 2$	D_6	2^2	**2**	**2**	**12**

Z = Φ ×4 **Z_2 = ℜ** × 4 × 4 **G′** × 2

Sylow subgroups: [Q_8] × 9, [3^2]

Maximal subgroups: [Q_{24}] × 12, [**12 × 3**], [**36.10**] × 2

72.39 = ⟨A^2, B^9, C^2, D^2, DB = B^5D, DC = ACD⟩

CLASS EQUATION: 72 = 1∗2 + 2∗17 + 18∗2 **#CC** = 21

ORDER EQUATION: 267∗2 = 2∗21 + 3∗2 + 4∗18 + 6∗6 + 9∗6 + 18∗18

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	2^2	**6**	**9**	**6 × 2**	**18**	**12 × 3**	**G**
G/H	**G**	$D_{18} \times 2$	**24.11**	D_{18}	$D_6 \times 2$	D_8	D_6	2^2	**2**	**1**

Z = ℜ **Z_2** **Φ** **G′** × 3

Sylow subgroups: [D_8] × 9, [**9**]

Maximal subgroups: [**24.11**] × 3, [**12 × 3**] × 3

72.40 = ⟨A^9, B^4, C^2, CA = A^−1C, CB = B^−1C⟩

CLASS EQUATION: 72 = 1∗2 + 2∗17 + 18∗2 **#CC** = 21

ORDER EQUATION: 371∗2 = 2∗37 + 3∗2 + 4∗2 + 6∗2 + 9∗6 + 12∗4 + 18∗6 + 36∗12

NORMAL SUBGROUPS:

H	**1**	**2**	**4**	**6**	**9**	**6 × 2**	**6 × 3**	**36**	**$D_{18} \times 2$**	**$D_{9,4}$**	**G**
G/H	**G**	**$D_{18,2}$**	**D_{18}**	**$D_6 \times 2$**	**D_8**	**D_6**	**2^2**	**2**	**2**	**2**	**1**
		Z = ℜ	**Z_2**	**Φ**			**G′**				

Sylow subgroups: [**D_8**] × 9, [**9**]

Maximal subgroups: [**D_{24}**] × 3, [**36**], [**$D_{18} \times 2$**] [**$D_{9,4}$**]

72.41 = ⟨A^9, B^4, C^2 = B^2, CA = A^−1C, CB = B^−1C⟩

CLASS EQUATION: 72 = 1∗2 + 2∗17 + 18∗2 **#CC** = 21

ORDER EQUATION: 407∗2 = 2 + 3∗2 + 4∗38 + 6∗2 + 9∗6 + 12∗4 + 18∗6 + 36∗12

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	**4**	**6**	**9**	**6 × 2**	**6 × 3**	**6^2**	**$D_{9,4}$**	**G**
G/H	**G**	**$D_{18} \times 2$**	**Q_{24}**	**D_{18}**	**$D_6 \times 2$**	**Q_8**	**D_6**	**2^2**	**2**	**2**	**1**
		Z		**Z_2 = ℜ**	Φ			**G′**		× 2	

Sylow subgroups: [**Q_8**] × 9, [**9**]

Maximal subgroups: [**Q_{24}**] × 3, [**6^2**], [**$D_{9,4}$**] × 2

72.42 = ⟨A^2, B^3, C^3, D^2, E^2, DB = B^−1D, EC = C^−1E, ED = ADE⟩

CLASS EQUATION: 72 = 1∗2 + 2∗4 + 4∗2 + 6∗6 + 18 **#CC** = 15

ORDER EQUATION: 157∗2 = 2∗13 + 3∗8 + 4∗18 + 6∗32

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	**6**	**3^2**	**$D_6 \times 2$**	**6 × 3**	**$D_6 \times 6$**	**36.10**	**G**
G/H	**G**	**$D_6 \times D_6$**	**24.11**	**$D_6 \times 2$**	**D_8**	**D_6**	**2^2**	**2**	**2**	**1**
		Z = ℜ = Φ	**× 2**	**× 2**		**× 2**	**G′**	**× 2**		

Sylow subgroups: [**D_8**] × 9, [**3^2**]

Maximal subgroups: [**24.11**] × 6, [**$D_6 \times 6$**] × 2, [**36.10**]

72.43 = ⟨A³, B³, C⁴, D², CA = A⁻¹C, DB = B⁻¹D, DC = C⁻¹D⟩

CLASS EQUATION: 72 = 1∗2 + 2∗4 + 4∗2 + 6∗6 + 18 **#CC** = 15

ORDER EQUATION: 181∗2 = 2∗25 + 3∗8 + 4∗6 + 6∗20 + 12∗12

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	**3**	**6**	**3^2**
G/H	**G**	**$D_6 \times D_6$**	**D_{24}**	**24.11**	**$D_6 \times 2$**	**D_8**

Z = ℜ = Φ

H	**$D_{3,4}$**	**$D_6 \times 2$**	**6×3**	**$D_{3,4} \times 3$**	**$D_6 \times 6$**	**18.4×2**	**G**
G/H	**D_6**	**D_6**	**2^2**	**2**	**2**	**2**	**1**

G′

Sylow subgroups: [**D_8**] × 9, [**3^2**]

Maximal subgroups: [**24.11**] × 6, [**$D_{3,4} \times 3$**], [**$D_6 \times 6$**], [**18.4×3**]

72.44 = ⟨A³, B³, C⁴, D² = C², CA = A⁻¹C, DB = B⁻¹D, DC = C⁻¹D⟩

CLASS EQUATION: 72 = 1∗2 + 2∗4 + 4∗2 + 6∗6 + 18 **#CC** = 15

ORDER EQUATION: 241∗2 = 2 + 3∗8 + 4∗30 + 6∗8 + 12∗24

NORMAL SUBGROUPS:

H	**1**	**2**	**3**	**6**	**3^2**	**$D_{3,4}$**	**6×3**	**6^2**	**36.10**	**G**
G/H	**G**	**$D_6 \times D_6$**	**Q_{24}**	**$D_6 \times 2$**	**Q_8**	**D_6**	**2^2**	**2**	**2**	**1**

Z = ℜ = Φ × 2 × 2 × 2 **G′** × 2

Sylow subgroups: [**Q_8**] × 9, [**3^2**]

Maximal subgroups: [**24.13**] × 6, [**6^2**] × 2, [**36.10**]

72.45 = ⟨A³, B³, C⁸, CA = ABC, CB = AC⟩

CLASS EQUATION: 72 = 1 + 8 + 9∗7 **#CC** = 9

ORDER EQUATION: 201∗2 = 2∗9 + 3∗8 + 4∗18 + 8∗36

NORMAL SUBGROUPS:

H	**1**	**3^2**	**18.4**	**36.14**	**G**
G/H	**G**	**8**	**2^2**	**2**	**1**

G′

Z = ℜ = Φ = [1]

Sylow subgroups: [**8**] × 9, [**3^2**]

Maximal subgroups: [**8**] × 9, [**36.14**]

72.46 = ⟨A³, B², C², D³, E², DB = BCD, DC = BD, EA = A⁻¹E⟩

CLASS EQUATION: 72 = 1 + 2 + 3∗2 + 4∗2 + 6 + 8∗2 + 9 + 12∗2 **#CC** = 12

ORDER EQUATION: 144∗2 = 2∗15 + 3∗26 + 6∗30

NORMAL SUBGROUPS:

H	**1**	**3**	**2^2**	**D_6**	**6 × 2**	**A_4**	**$D_6 \times 2^2$**	**$A_4 \times 3$**	**G**
G/H	**G**	**$A_4 \times 2$**	**$D_6 \times 3$**	**A_4**	**6**	**D_6**	**3**	**2**	**1**

Z = ℵ = Φ **G′**

Sylow subgroups: [2^3] × 3, [3^2] × 4

Maximal subgroups: [$A_4 \times 2$] × 3, [$D_6 \times 2^2$], [$A_4 \times 3$]

72.47 = ⟨A³, B³, C², D⁴, CA = BC, CB = AC, DB = AD⟩

CLASS EQUATION: 72 = 1 + 4∗2 + 6∗2 + 9 + 12∗2 + 18 **#CC** = 9

ORDER EQUATION: 141∗2 = 2∗21 + 3∗8 + 4∗18 + 6∗24

NORMAL SUBGROUPS:

H	**1**	**3^2**	**18.4**	**$D_6 \times D_6$**	**36.14**	**G**
G/H	**G**	**D_8**	**2^2**	**2**	**2**	**1**

Z = ℵ = Φ **G″** **G′** × 2

Sylow subgroups: [D_8] × 9, [3^2]

Maximal subgroups: [D_8] × 9, [$D_6 \times D_6$] × 2, [**36.14**]

72.48 = ⟨A³, B³, C⁴, D² = C², CA = BC, CB = A⁻¹C, DA = AB², DB = A²B²D, DC = C⁻¹D⟩

CLASS EQUATION: 72 = 1 + 8 + 9 + 18∗3 **#CC** = 6

ORDER EQUATION: 129∗2 = 2∗9 + 3∗8 + 4∗54

NORMAL SUBGROUPS:

H	**1**	**3^2**	**18.4**	**36.14**	**G**
G/H	**G**	**Q_8**	**2^2**	**2**	**1**

Z = ℵ = Φ **G″** **G′** × 3

Sylow subgroups: [Q_8] × 9, [3^2]

Maximal subgroups: [Q_8] × 9, [**36.14**] × 3

72.49 = ⟨A³, B², C², D³, E², DB = BCD, DC = BD, EA = A⁻¹E, EC = B⁻¹CE, ED = D⁻¹E⟩

CLASS EQUATION: 72 = 1 + 2 + 3 + 6 + 8∗3 + 18∗2 **#CC** = 9
ORDER EQUATION: 114∗2 = 2∗21 + 3∗26 + 4∗18 + 6∗6
NORMAL SUBGROUPS:

H	**1**	**3**	2^2	**6 × 2**	A_4	$A_4 \times 3$	**G**
G/H	**G**	S_4	**18.4**	D_6	D_6	**2**	**1**

G″ × 3 **G′**

Sylow subgroups: [D_8] × 9, [3^2]
Maximal subgroups: [**24.11**] × 3, [S_4] × 9, [$A_4 \times 3$], [**18.4**] × 4

72.50 = ⟨A², B², C⁹, E², CA = ABC, CB = AC, DA = BD, DB = AD, DC = C⁻¹D⟩

CLASS EQUATION: 72 = 1 + 2 + 3 + 6 + 8∗3 + 18∗2 **#CC** = 9
ORDER EQUATION: ∗2 = 2∗21 + 3∗2 + 4∗18 + 6∗6 + 9∗24
NORMAL SUBGROUPS:

H	**1**	**3**	2^2	**6 × 2**	**36.8**	**G**
G/H	**G**	$Q_8 \times 3$	D_{18}	D_6	**2**	**1**

Z = 𝔄 **Φ** **G″** **G′**

Sylow subgroups: [D_8] × 9, [**9**] × 4
Maximal subgroups: [**24.11**] × 3, [**36.8**], [D_{18}] × 4